

# The pressure drop in microtubes and the correlation development

Yun Wook Hwang<sup>a</sup>, Min Soo Kim<sup>b,\*</sup>

<sup>a</sup> Energy System Research Center, Korea Institute of Machinery and Materials, Daejeon 305-343, Republic of Korea

<sup>b</sup> School of Mechanical and Aerospace Engineering, Seoul National University, Seoul 151-744, Republic of Korea

Received 20 January 2005; received in revised form 26 August 2005

Available online 19 January 2006

## Abstract

This study investigated the pressure drop characteristics in microtubes using R-134a as a test fluid. The test tubes were the circular stainless steel tubes with inner diameters of 0.244, 0.430, and 0.792 mm. Although some of the existing studies reported the early flow transition at the Reynolds number of less than 1000, it was not found in the single-phase flow pressure drop tests. The conventional theory predicted the friction factors well within an absolute average deviation of 8.9%. The two-phase flow pressure drop increased with increasing quality, increasing mass flux, and decreasing tube diameter. The existing correlations failed to predict the two-phase friction multipliers in the microtubes of this study. A new correlation to predict the two-phase flow pressure drop in microtubes was developed in the form of the Lockhart–Martinelli correlation. It includes the effect of the tube diameter, surface tension effect, and the effect of the Reynolds number on the two-phase flow pressure drop in microtubes. The new correlation developed in this study predicted the experimental data within an absolute average deviation of 8.1%.

© 2005 Elsevier Ltd. All rights reserved.

**Keywords:** Microtube; Pressure drop; Friction factor; Two-phase frictional multiplier

## 1. Introduction

A lot of attention has been paid to the transport phenomena in the microgeometries because microfabrication enables us to manufacture the flow channels ranging from a few micrometers to a few hundred micrometers. Microfabrication technique can be applied in making the microchannels, and also can be used in manufacturing the microsystems like microcooling devices, microturbines, and so on. The fluid flow in the microchannels attracts people's attention because it has a variety of advanced applications in the microelectro-mechanical systems (MEMS), electronic cooling systems, bioengineering, and others. In spite of the brilliant development of microfabrication technique, the transport phenomenon in the microgeometries is still one of the unexplored fields because of the many difficulties in the experiments and the analyses.

The studies on the pressure drop in microchannels have been focused on single-phase flow [1–10]. When frictional characteristics are analyzed in terms of friction constant ( $f \cdot Re$ ), previous studies are divided into three groups. The first is by Wu and Little [1], Mala and Li [7], and Tu and Hrnjak [9]. They predicted greater values of friction constant ( $f \cdot Re$ ) for the microchannels than for the conventional channels. The second is by Choi et al. [3] and Yu et al. [11], which predicted constant lower values than those for conventional channels. The last is characterized by Peng et al. [4], which showed a different trend; in their test cases, friction constant ( $f \cdot Re$ ) decreased with an increase in the Reynolds number. Also they concluded that the value of friction constant is the greatest for the largest microchannels ( $D_h$ ) and the lowest for the smallest ones while the slopes of the curves between friction constant and the Reynolds number are identical as  $Re^{-0.98}$ .

Table 1 shows the experimental results of the previous studies on friction factors in the microchannels or the microtubes. Some studies gave the result that the experimental friction factors are greater than the theoretical

\* Corresponding author. Tel.: +82 2 880 8362; fax: +82 2 883 0179.  
E-mail address: [minskim@snu.ac.kr](mailto:minskim@snu.ac.kr) (M.S. Kim).

**Nomenclature**

$C$	$C$ coefficient defined by Lockhart and Martinelli [16]	$\varepsilon$	surface roughness ( $\mu\text{m}$ )
$C_0$ – $C_3$	constants used in Eq. (17)	$\mu$	viscosity ( $\text{Pa s}$ )
$C_f$	friction coefficient	$\rho$	density ( $\text{kg m}^{-3}$ )
$D_i$	inner diameter (mm)	$\sigma$	surface tension ( $\text{N m}^{-1}$ )
$-(dp/dz)$	pressure drop gradient ( $\text{kPa m}^{-1}$ )	$\sigma_{aa}$	absolute average deviation (%)
$f$	friction factor	$\phi$	two-phase frictional multiplier
$G$	mass flux ( $\text{kg m}^{-2} \text{s}^{-1}$ )	<i>Subscripts</i>	
$I$	current (A)	cal	calculated values
$i$	enthalpy ( $\text{kJ kg}^{-1}$ )	f	liquid phase
$L$	length (mm)	fg	phase change between liquid phase and vapor phase
$L_C$	Laplace constant defined in Eq. (15)	g	vapor phase
$\dot{m}$	mass flow rate ( $\text{g h}^{-1}$ )	in	inlet
$N_{\text{conf}}$	confinement number defined in Eq. (16)	Lo	liquid-only
$Re$	Reynolds number	m	mixture
$X$	Martinelli parameter	meas	measured values
$U$	velocity ( $\text{m s}^{-1}$ )	p	preheater
$V$	voltage (V)	pred	predicted values
$v$	specific volume ( $\text{m}^3 \text{kg}^{-1}$ )	sp	single-phase
$x$	quality	t	test section
<i>Greek symbols</i>		tp	two-phase
$\Delta p$	pressure drop (kPa)		
$\delta$	deviation (%)		

values [1,7,9]. Others gave different results that the experimental friction factors are less than the theoretical values [3,11]. Pfahler et al. [2], Lee and Lee [12], and Liu and Garimella [10] concluded that the friction factors in the microchannels or the microtubes satisfactorily agree with the theoretical prediction. Peng et al. [4] and Peng and Peterson [6] showed that the aspect ratio of the microchannel determines whether the friction factor is greater than the theoretical value or not.

There have been, if any, few studies on two-phase flow pressure drop in microgeometry. It is difficult to analyze and predict the pressure drop of the two-phase flow

because there are three factors associated with pressure drop; fluid friction, phase change, and gravity. The studies in microgeometry are mainly for the rectangular channels and there are few studies for the circular tubes with the diameter of less than 1 mm.

Air and water have been adopted as the test fluids in most of the studies about the two-phase pressure drop because it is easy to adjust the velocities of the liquid phase and the vapor phase with air and water. Two-phase flow of two components is involved here, however, this is dissimilar to the two-phase flow of one component. It is worthwhile to handle practical refrigerants, which are being used as working fluids of actual heat pumps or refrigerators in the two-phase flow pressure drop studies.

In this study, the single-phase flow pressure drop and the two-phase flow pressure drop in the microtubes are experimentally investigated and theoretically analyzed using R-134a as a test fluid. Also, based on the measured experimental results, the correlation to predict two-phase flow pressure drop in microtubes is proposed in the form of Lockhart–Martinelli type correlation.

## 2. Experiments

### 2.1. Experimental test rig

Fig. 1 shows the schematic view of experimental test rig built for the test. It has two syringe pumps, filter,

Table 1  
The conflicting results between the existing studies about the friction factor of the single-phase flow

Investigator	$f > f_{\text{theory}}$	$f \cong f_{\text{theory}}$	$f < f_{\text{theory}}$
Wu and Little [1]	✓		
Pfahler et al. [2]		✓	
Choi et al. [3]			✓
Peng et al. [4]	✓		✓
Yu et al. [11]			✓
Peng and Peterson [6]	✓		✓
Mala and Li [7]	✓		
Lee and Lee [12]		✓	
Faghri and Turner [8]		✓	
Tu and Hrnjak [9]	✓		
Liu and Garimella [10]		✓	

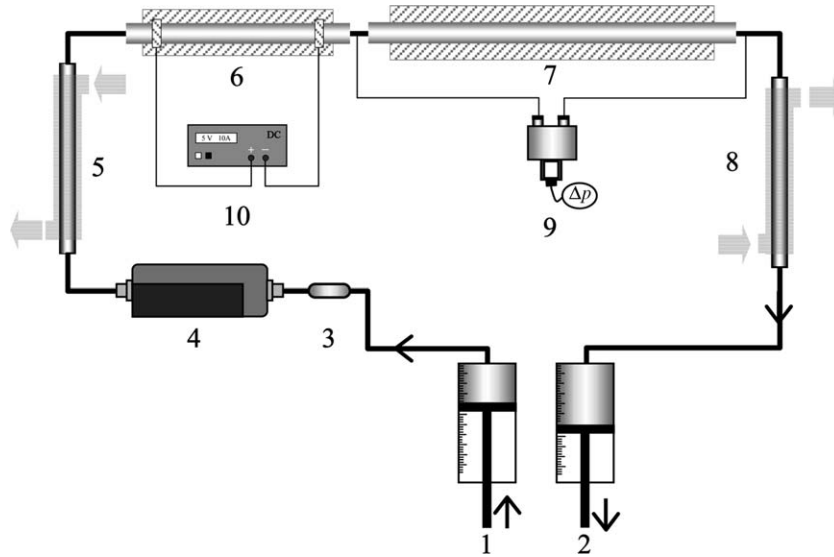


Fig. 1. Experimental test rig for the pressure drop test (1: syringe pump 1, 2: syringe pump 2, 3: filter, 4: thermal mass flow meter, 5: subcooler, 6: preheater, 7: test section, 8: condenser, 9: differential pressure transducer, 10: DC power supply).

subcooler, mass flow meter, preheater, test section, and the condenser. The syringe pump is known to have two important functions to discharge the liquid phase at constant flow rate and keep the system pressure constant. Therefore, syringe pump 1 in Fig. 1 is used to discharge the liquid refrigerant into the test section at a constant flow rate and syringe pump 2 is used to keep the system pressure in the test rig constant. The thermal mass flow meter measures the mass flow rate of refrigerant flowing into the test section. Precise control of the mass flow rate is made possible with the syringe pumps and the thermal mass flow meter. The subcooler and the preheater are installed in order to control the quality of the refrigerant at the inlet of the test section. The refrigerant which comes out of the subcooler goes into the preheater. Direct heating method is employed for providing the heat to the refrigerant in the preheater. The DC power supply (HP 6673A) is used to provide the accurate amount of electricity to the refrigerant. The refrigerant exiting the test section as a two-phase fluid is condensed to the single-phase state in the condenser. The liquid phase refrigerant goes into the other syringe pump 2 which maintains the system pressure constant.

## 2.2. Test sections

Three kinds of the microtubes are used to investigate the pressure drop characteristics in microtubes. Table 2 shows the specifications about the test tubes.  $L_{sp}$  represents the

Table 2  
Specifications of the test sections for the pressure drop test

Test tube	$D_i$ (mm)	$\varepsilon$ ( $\mu\text{m}$ )	$L_{sp}$ (mm)	$L_{tp}$ (mm)
A	0.244	0.397	400	60
B	0.430	0.486	400	180
C	0.792	0.341	462	462

length of the test tube for the single-phase pressure drop test and  $L_{tp}$  represents the length for the two-phase pressure drop test. The reason why the test tubes have different lengths is that there is a great difference in pressure drop between the single-phase flow and the two-phase flow. Two layers of the ceramic fiber insulator and the rubber foaming insulator thermally insulate the test section to minimize the heat transfer between the refrigerant flowing in the test section and the ambient atmosphere. The heat addition to the test fluid flowing through the test section was estimated to be very small and would not cause the quality change in the two-phase pressure drop tests which could have meaningful effect on the characteristics of the pressure drop in microtubes.

## 2.3. Data reduction

One of the best ways to investigate the characteristics of the single-phase flow pressure drop is to analyze the relationship between the friction factor and the Reynolds number. The friction factors are calculated and compared to the theoretically predicted values. The friction factor is given as the following equation (1):

$$f = \left( \frac{\pi^2 \cdot \rho_f \cdot (\Delta p/L)}{8 \cdot \dot{m}^2} \right) \cdot D_i^5 \quad (1)$$

It is valid when the flow is fully-developed in the tube.

Reynolds number,  $Re$ , is defined as the following equation (2):

$$Re = \frac{4\dot{m}}{\pi D_i \mu_f} \quad (2)$$

In the two-phase flow pressure drop test, the Martinelli parameter and the two-phase frictional multiplier are used to investigate the characteristics of the two-phase flow

pressure drop in the microtubes. The Martinelli parameter, which is shown in Eq. (3), is the ratio of the pressure drops of the liquid phase flow and the vapor phase flow.

$$X^2 = \left(\frac{dp}{dz}\right)_f / \left(\frac{dp}{dz}\right)_g \quad (3)$$

The two-phase frictional multiplier is defined as Eq. (4). It is the ratio of the pressure gradients of the two-phase flow and the liquid phase flow.

$$\phi_f^2 = \left(\frac{dp}{dz}\right)_{tp} / \left(\frac{dp}{dz}\right)_f \quad (4)$$

The pressure drops of the liquid phase flow and the vapor phase flow are expressed as Eqs. (5) and (6), respectively.

$$-\left(\frac{dp}{dz}\right)_f = \frac{2 \cdot C_{f,f} \cdot (G \cdot (1-x))^2}{D_i \cdot \rho_f} \quad (5)$$

$$-\left(\frac{dp}{dz}\right)_g = \frac{2 \cdot C_{f,g} \cdot (G \cdot x)^2}{D_i \cdot \rho_g} \quad (6)$$

The mass quality at the inlet of the test section is the same as that at the outlet of the preheater because the inlet of the test section is the same location as the outlet of the preheater. The heat provided in the preheater determines the mass quality at the inlet of the test section. The heat is supplied by the electricity in the direct heating method of this study. The refrigerant flowing into the preheater should be in the subcooled state for the calculation of the mass quality, but it becomes saturated when entering the test section. The mass quality at the inlet of the test section is expressed as Eq. (7).

$$x_{i,in} = \frac{(V_p \cdot I_p) / \dot{m} + i_{f,p,in} - i_{f,t,in}}{i_{fg,t,in}} \quad (7)$$

The mass quality does not change a lot along the test section because no heat is provided to the refrigerant flowing in the test section. However, the mass quality at the inlet of the test section is adjusted by the preheater and it is used in calculating the Martinelli parameters and the two-phase multipliers.

The thermophysical properties should be known to calculate the Martinelli parameter, two-phase multiplier, quality, and etc. REFPROP [13], which is widely used in calculating the thermophysical properties of refrigerants, was used.

The uncertainty analysis was done for the experimental friction factors and two-phase frictional multipliers according to the procedures described by Coleman and Steele [14] and the average uncertainties of the friction factors and the two-phase frictional multipliers were 4.6% and 8.4%, respectively.

2.4. Test condition

Table 3 shows the test conditions in this study. For the single-phase flow pressure drop tests, the ratios of the

Table 3  
Test conditions for the pressure drop tests

Test tube	Single-phase flow	Two-phase flow	
	$\dot{m}$ (g/h)	$G$ (kg/m <sup>2</sup> s)	Quality
A	110–1210	480, 730, 950	0.10–0.95
B	137–1098	270, 510, 900	0.08–0.84
C	110–1184	140, 270, 470	0.11–0.88

length and the inner diameter are all over 500, which confirms that the flows in the pressure drop test can be presumed to be fully developed. The controlling parameter is the mass flow rate that is varied with the syringe pump 1.

For the two-phase flow pressure drop tests, the mass flux and the quality were controlled by the syringe pump 1 and the preheater. Their ranges are listed in Table 3.

3. Experimental results and discussion

3.1. Single-phase flow pressure drop

Fig. 2 shows the pressure drop per unit length versus Reynolds number in the microtubes, which is named as test tube A, test tube B, and test tube C, respectively. All the pressure drop tests have been done under the adiabatic condition. Therefore, all the data show frictional pressure

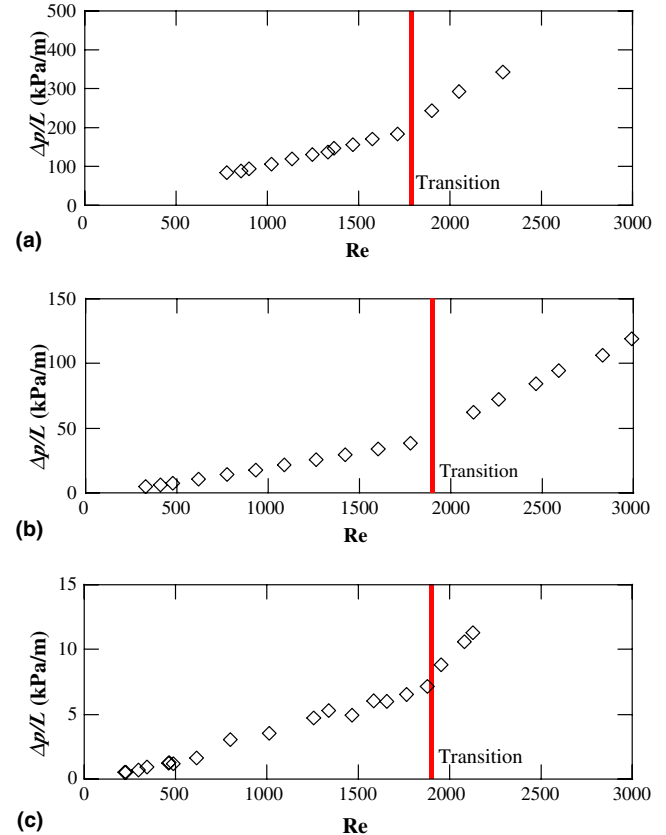


Fig. 2. The pressure drop per unit length versus Reynolds number. (a) Test tube A ( $D_i = 0.244$  mm); (b) test tube B ( $D_i = 0.430$  mm); (c) test tube C ( $D_i = 0.792$  mm).

drop. As easily expected, pressure drop per unit length increases with increasing Reynolds number. It is because the friction force increases due to the increase of the average velocity of fluid and the shear stress on the surface. This phenomenon is exactly the same as in the conventional-sized tubes. In the low Reynolds number region, the pressure drop increases linearly with the Reynolds number. At Reynolds number of slightly less than 2000, however, pressure drop starts to increase sharply, not linearly. This means that the flow experiences a transition from laminar to turbulent flow. Wu and Little [1] carried out an experimental study with nitrogen in the trapezoidal microchannels made of silicon and glass and presented the result that flow transition occurs at  $Re \approx 400$ . Peng et al. [4] showed in their study that the transition from laminar flow to turbulent flow occurs at  $Re \approx 700$  for  $D_h > 267 \mu\text{m}$ . They explained that the intensity of the velocity fluctuation due to the inertial force and viscosity required to initiate the turbulence is smaller than that for the normal flow in large or macroducts, and any wall effect would easily and quickly penetrate into the main stream fluid zone and influence the entire flow. Mala and Li [7] made an experimental investigation on flow characteristics with water in the circular microtubes made of silica and stainless steel. They concluded that the experimental results indicated significant departure of flow characteristics from the predictions by the conventional theory for microtubes with smaller diameters and there might be an early transition from laminar to turbulent flow mode at  $Re > 300\text{--}900$ . They mentioned that the presence of surface roughness affects the laminar velocity profile and decreased the transitional Reynolds number. However, Lee and Lee [12] did not find the early transition in the single-phase flow pressure drop test with the microchannels with a hydraulic diameter of 0.784 mm. Faghri and Turner [8] investigated the effect of relative surface roughness in the microchannels made of (100) and (101) silicon wafer. They fabricated various microchannels having different relative surface roughness and tested them with nitrogen and helium. They could not find a significant effect of relative surface roughness in the range of  $\varepsilon/H$  from 0.001 to 0.06. Liu and Garimella [10] showed that at  $Re \approx 2000$ , the experimental results start to deviate from the laminar predictions, indicating the onset of the transition. In this study, flow transition is also identified, but the transition is not as early as in Wu and Little [1], Peng and Peterson [6], and Mala and Li [7]. The experimental results show that the flow transition for the test tubes of the diameter of down to 0.244 mm occurs at  $Re \approx 2000$  or less. Obot [15] concluded that there is hardly any evidence to support the occurrence of transition to turbulence for  $Re < 1000$  and the early transition at Reynolds number as low as 400 in Wu and Little [1] and 700 in Peng et al. [4] can be ascribed to possible experimental errors.

Friction factors are plotted against Reynolds number for test tubes A–C in Fig. 3. The trend of the friction factors versus Reynolds numbers is so much the same as for the conventional-sized tubes. According to the classical

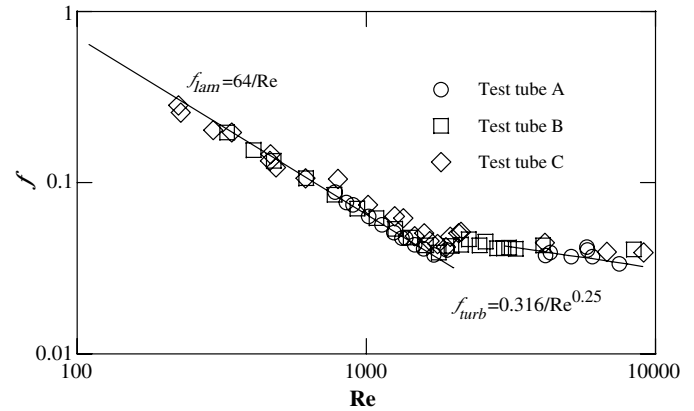


Fig. 3. The friction factors versus the Reynolds numbers for the test tubes in this study.

theory about laminar flow, the product of friction factor and Reynolds number is 64 for circular tubes. As seen in Fig. 3, test results are in a good agreement with a classical theory within an absolute average deviation of 8.9%. As described in Table 1, there have been controversial results about whether the conventional theory can predict friction factors in microtubes. This study supports that the pressure drop and the friction factors in microtubes of a diameter down to about 0.2 mm are very well predicted by the conventional theory.

The accuracy of friction factors is entirely dependent on the measurement of the parameters related to the single-phase pressure drop test. Eq. (1) shows that the most important parameter of them is the inner diameter of the microtube because friction factor is directly proportional to the inner diameter to the fifth power. It also means that the error in the measurement of the inner diameter can lead to the great uncertainty of the friction factor. In spite of its importance, it is not easy to measure the accurate diameter for the microtubes or the height and the width of the microchannels. In this study, the 10 sections for each microtube have been used for the measurement of the inner diameter with SEM (scanning electron microscope) which has a function to measure the diameter, length, etc., and then the average value of the measured inner diameters was used for calculating the friction factors.

As seen in Eq. (1), the other parameters are said to be easier to measure accurately. Therefore, the departure from the theoretical predictions found in the previous studies is likely to be due to less accurate measurement of the height and the width of the microchannels or the diameter for the microtubes.

### 3.2. Two-phase flow pressure drop

Fig. 4(a) shows the pressure drop per unit length of the test tube A versus mass quality. As easily expected, greater mass quality causes greater pressure drop because mean specific volume increases with increasing mass quality and thus the average velocity of refrigerant goes up.

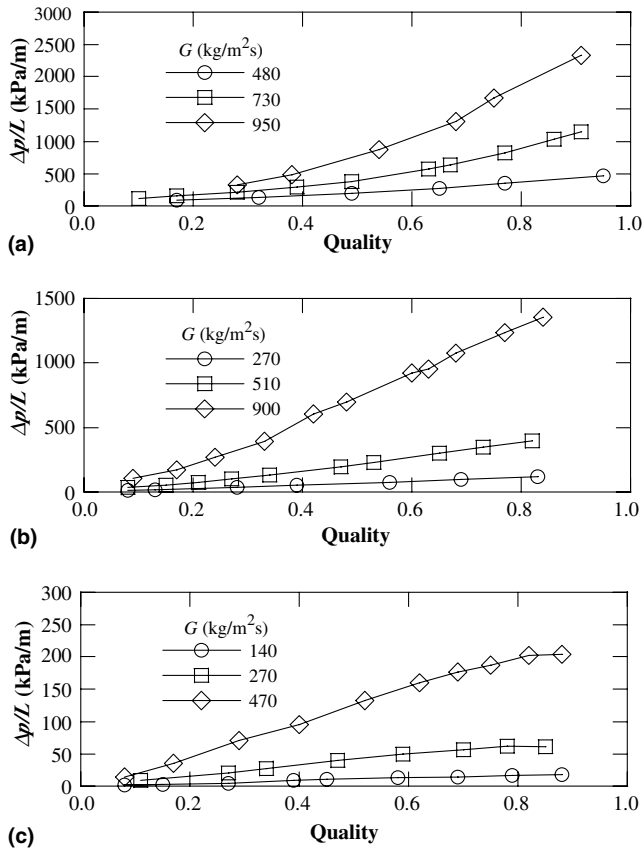


Fig. 4. The frictional two-phase pressure drop per unit length of the test tube. (a) Test tube A ( $D_i = 0.244$  mm); (b) test tube B ( $D_i = 0.430$  mm); (c) test tube C ( $D_i = 0.792$  mm).

As seen in Fig. 4(a), the effect of mass flux on the two-phase flow pressure drop is very significant because mass flux directly causes the friction force of fluid flow against tube wall. Eqs. (8) and (9) show the change in mean velocity on which the pressure drop depends with increasing mass flux and mass quality. Fig. 4(b) and (c) is the measured pressure drop data for the test tubes B and C, respectively and their trends are the same as that of the test tube A.

$$v_m = (1 - x) \cdot v_f + x \cdot v_g \tag{8}$$

$$U_m = G \cdot v_m \tag{9}$$

The experimental two-phase pressure drop data were reduced to the two-phase frictional multipliers and then compared with several existing correlations. They are the

correlations by Lockhart and Martinelli [16], Moriyama et al. [17], Mishima and Hibiki [18], and Lee and Lee [12]. There are other correlations to predict the two-phase flow pressure drop for larger-sized tubes, which are excluded for comparison with the experimental results in this study.

The absolute average deviations expressed in Eq. (10) are tabulated in Table 4.

$$\sigma_{aa} = \frac{1}{N} \left[ \sum_{k=1}^N \left| \frac{\phi_{f,pred,k}^2 - \phi_{f,meas,k}^2}{\phi_{f,meas,k}^2} \right| \right] \times 100(\%) \tag{10}$$

Table 4 also shows that the deviation increases with decreasing tube diameter and the Mishima and Hibiki [18] correlation predicts the experimental data better than the others. The Mishima and Hibiki [18] correlation was developed with the circular tubes with an inner diameter of 1–4 mm. Although Lee and Lee [12] correlation and Moriyama et al. [17] correlation were developed for rectangular microchannels having smaller hydraulic diameter than Mishima and Hibiki [18] correlation, they poorly predict the two-phase frictional multipliers. It means that the two-phase flow patterns in microcircular tubes are different from those in rectangular microchannels and the difference in flow patterns has an influence on the two-phase pressure drop. Fig. 5 shows the possible liquid film distribution in a circular microtube and a rectangular microchannel, respectively. Although they have the same hydraulic diameter, the perimeter of the microchannel is greater than that of the microtube. If the fluid in the same quality is flowing at the same velocity, the liquid film thickness should be different because the perimeter is different. The difference in liquid film distribution can affect two-phase flow pressure drop as well as two-phase flow heat transfer. Lee and Lee [12] correlation and Moriyama et al. [17] correlation were developed for the rectangular microchannels with the low

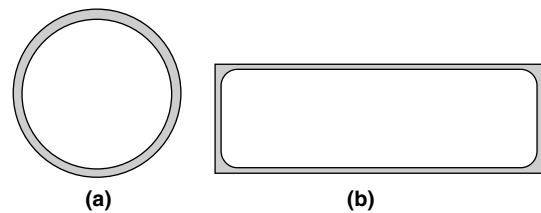


Fig. 5. The possible liquid film distribution during evaporation. (a) Circular microtube; (b) rectangular microchannel.

Table 4  
The comparison of the experimental results with the existing correlations

Correlation	$\sigma_{aa}$ (%)		
	Test tube A ( $D_i = 0.244$ mm)	Test tube B ( $D_i = 0.430$ mm)	Test tube C ( $D_i = 0.792$ mm)
Lockhart and Martinelli [16]	182.6	82.7	59.0
Moriyama et al. [17]	255.1	149.7	136.6
Mishima and Hibiki [18]	15.5	23.0	23.8
Lee and Lee [12]	117.4	76.5	52.4

aspect ratio of 0.02–0.2 and 0.00023–0.00326, respectively. It is known that the rectangular channel with the low aspect ratio less than about 0.45 is subject to higher pressure drop than the circular tube having the same hydraulic diameter. Therefore, Lee and Lee [12] correlation and Moriyama et al. [17] correlation are likely to overpredict two-phase flow pressure drop in circular tubes and rectangular channels with high aspect ratio because they were developed with rectangular channels having low aspect ratio of less than 0.2. They fail to predict the two-phase frictional multiplier for the two-phase flow in circular microtubes even if the hydraulic diameter is used in their correlation.

Although the Mishima and Hibiki [18] correlation predicts the experimental data better than the others, it has a specific tendency to underpredict the experimental data at small Martinelli parameters. As shown in Fig. 6, the deviation, which is defined in Eq. (11), is greater than +20% or less than –20% for 46% of the experimental two-phase frictional multipliers.

$$\delta = \left( \frac{\phi_{f,\text{pred},k}^2 - \phi_{f,\text{meas},k}^2}{\phi_{f,\text{meas},k}^2} \right) \times 100(\%) \quad (11)$$

This means that Mishima and Hibiki [18] correlation also fails to predict the two-phase frictional multipliers for the two-phase flow in circular microtubes.

### 3.3. Two-phase flow pressure drop correlation

Empirical correlations based upon experimental data are requisites for designing the heat exchangers such as evaporators and condensers. This cannot be applied only to heat exchangers, but also to whole thermodynamic systems. Few correlations which can predict the two-phase pressure drop in microtubes have been developed until now. In this study, an experiment-based correlation for the two-phase flow pressure drop in microtubes is proposed.

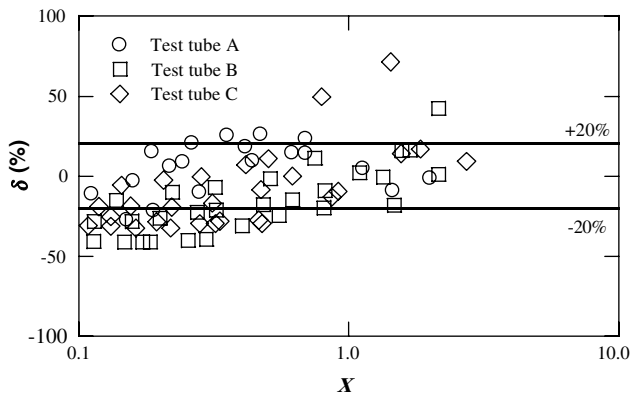


Fig. 6. The average deviation between the experimental two-phase frictional multiplier and the two-phase frictional multiplier calculated with Mishima and Hibiki [18] correlation with the respect to the variation of the Martinelli parameter.

Several correlations to predict two-phase flow pressure drop have been developed since Lockhart and Martinelli [16] first correlated the two-phase frictional multipliers,  $\phi_f^2$ ,  $\phi_g^2$ , as a function of the parameter  $X$ . Some researchers considered two-phase multipliers independent of mass flux, and others used two-phase multipliers dependent on mass flux.

Lockhart and Martinelli [16] showed that the parameters,  $\phi_f^2$ ,  $\phi_g^2$ , can be correlated to the parameter,  $X$ , by the form depicted in Eqs. (12) and (13). The coefficient  $C$  in Eqs. (12) and (13) is determined based on the flows of liquid phase and vapor phase.

$$\phi_f^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \quad (12)$$

$$\phi_g^2 = 1 + C \cdot X + X^2 \quad (13)$$

Many correlations have been developed in the form of Lockhart–Martinelli correlation that related the two-phase multipliers to Lockhart–Martinelli parameter,  $X$ . It is because the Lockhart–Martinelli type correlation is easier to develop and use than others.

$$-\left(\frac{dp}{dz}\right)_{\text{tp}} = -\left(\frac{dp}{dz}\right)_f + C \cdot \sqrt{-\left(\frac{dp}{dz}\right)_f} \sqrt{-\left(\frac{dp}{dz}\right)_g} - \left(\frac{dp}{dz}\right)_g \quad (14)$$

As expressed in Eq. (14), Lockhart and Martinelli considered that the two-phase pressure drop consists of three terms. The first term on the right-handed side is the pressure drop of the liquid phase. The second is the term that represents the interaction between the liquid phase and the vapor phase. The last term is the pressure drop of vapor phase. Eq. (14) becomes Eq. (12) when it is divided by  $-(dp/dz)_f$ .

The Lockhart and Martinelli [16] correlation has a constant  $C$ -coefficient which is independent of the flow geometry. It predicts well the two-phase frictional multipliers in larger-sized tubes. However, the Martinelli correlation did not consider the interaction factor between the liquid and the vapor phase. So, it gives satisfactory results about two the phase pressure drop in many engineering problems, but may fail in the flow through the microchannels like in this study. However, the Martinelli correlation method gives a very good guideline for developing a new correlation for the two-phase pressure drop. Moriyama et al. [17] correlation showed that Lockhart–Martinelli type correlation with  $C = 0$  fitted well with their experimental data and considered the liquid Reynolds number in their correlation. Mishima and Hibiki [18] correlation considers the effect of hydraulic diameter only on  $C$ -coefficient and neglects the flow velocity or surface tension which is known to be a dominant parameter in microgeometries. However, it predicts the experimental data in this study better than the other correlations because it was developed for circular tubes. Lee and Lee [12] developed the correlation that considers the flow velocity, surface tension, and channel

geometry. Although it considers nearly all parameters that influence the two-phase pressure drop, it predicts the experimental data well for rectangular channels, not for circular tubes. Their smallest test channel is 0.4 mm in height and 4–20 mm in width, where its smallest hydraulic diameter is 0.784 mm.

It is better that a new correlation will consider reasonable number of parameters related to the two-phase flow pressure drop in microgeometries. The first parameter considered is surface tension. The surface tension effect is dominant in small or microchannels when the Laplace constant ( $L_C$ , Eq. (15)) is greater than the hydraulic diameter. The diameters of the microtubes adopted in this study are greater than the Laplace constant. Therefore, the confinement number ( $N_{conf}$ , Eq. (16)), which was first defined by Kew and Cornwell [19], is included in a newly-developed correlation.

$$L_C = \sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}} \tag{15}$$

$$N_{conf} = \sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}} / D_i \tag{16}$$

It can consider increasing surface tension effect in microgeometries and the change of the tube diameter for the two-phase flow pressure drop.

The second parameter is the Reynolds number. Fig. 7 shows the dependence of the liquid-only Reynolds number, which is denoted by  $Re_{Lo}$ , on two-phase frictional multipliers.  $C$  in the ordinate axis is the value which makes the correlation satisfy the experimental results in Eq. (12), and  $X$  in the abscissa is the Martinelli parameter. Fig. 7 shows that  $C$ -coefficient is not constant but changes with respect to the variation of  $Re_{Lo}$  and  $X$ . Therefore,  $Re_{Lo}$  and  $X$  should be included in the new correlation.

$C$ -coefficient is newly defined in the form of Eq. (17). It considers the dominant parameters of  $Re_{Lo}$ ,  $X$ , and  $N_{conf}$ .

$$C = C_0 \cdot (Re_{Lo})^{C_1} \cdot X^{C_2} \cdot (N_{conf})^{C_3} \tag{17}$$

The constants of  $C_0$ ,  $C_1$ ,  $C_2$ , and  $C_3$  were calculated using the least error square method and are listed in Table 5. As

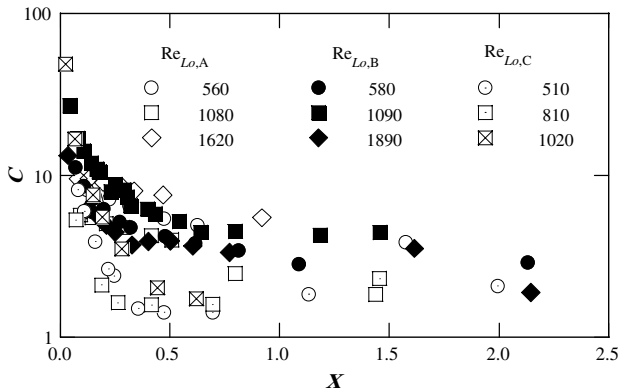


Fig. 7. The coefficient  $C$  versus Martinelli parameter for the test tubes in this study.

Table 5  
 $C_0$ ,  $C_1$ ,  $C_2$ , and  $C_3$  of  $C$ -coefficient in Eq. (17)

Parameter	Value
$C_0$	0.227
$C_1$	0.452
$C_2$	-0.320
$C_3$	-0.820

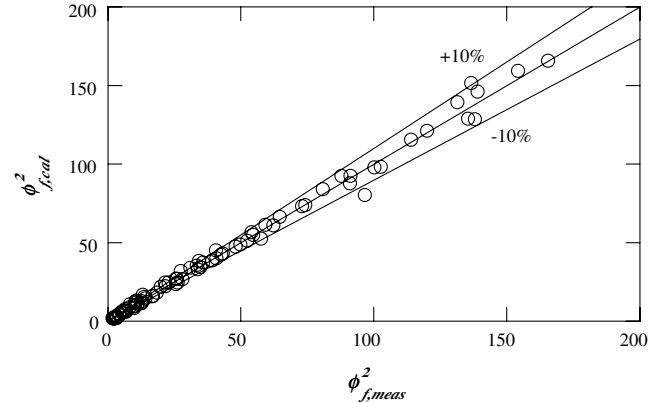


Fig. 8. The comparison of the calculated two-phase frictional multipliers with the experimental two-phase frictional multipliers.

shown in Fig. 8, the newly developed correlation predicts the experimental data quite well within an absolute average deviation of 8.1%.

#### 4. Conclusions

In this study, the characteristics about the pressure drop of the single-phase flow and the two-phase flow in the microtubes with the diameter from 0.244 mm to 0.792 mm have been investigated and the findings are as follows.

- (1) Early flow transition which has been reported in several previous studies is not found in this study. The onset of flow transition occurs at the Reynolds number of slightly less than 2000. Although there has been a controversy about whether the conventional theory about friction factor underpredicts the experimental data or overpredicts the experimental data, it generally predicts well the experimental data in this study.
- (2) The two-phase flow pressure drop increases with increasing quality, increasing mass flux, and decreasing inner diameter. However, the existing correlations predict poorly the experimental data because the surface tension effect is not normally considered, which is truly dominant in microtubes unlike conventional tubes. The previous correlations developed with the microchannels predict poorly the experimental data measured with the microtubes even though hydraulic diameter is used.



- (3) A new correlation to predict the two-phase flow frictional pressure drop is developed in the form of the Lockhart–Martinelli correlation. The parameters influencing on the two-phase flow frictional pressure drop in microtubes are included in the newly-developed correlation, which are the liquid-only Reynolds number, Martinelli parameter, and the confinement number. The new correlation predicts the experimental data within an absolute average deviation of 8.1%.

### Acknowledgements

This work was jointly supported by the Micro Thermal System Research Center and the BK21 Project of the Ministry of Education & Human Resources Development.

### References

- [1] P. Wu, W.A. Little, Measurement of friction factors for the flow of gases in very fine channels used for microminiature Joule–Thomson refrigerators, *Cryogenics* 23 (1983) 273–277.
- [2] J. Pfahler, J. Harley, H.H. Bau, J. Zemel, Liquid transport in micron and submicron channels, *J. Sens. Actuators* 21 (1990) 431–434.
- [3] S.B. Choi, R.F. Zemel, R.O. Warrington, Fluid flow and heat transfer in microtubes, *Micromech. Sens. Actuators Syst.* 32 (1991) 123–134.
- [4] X.F. Peng, G.P. Peterson, B.X. Wang, Frictional flow characteristics of water flowing through rectangular microchannels, *Exp. Heat Transfer* 7 (1994) 249–264.
- [5] X.F. Peng, G.P. Peterson, B.X. Wang, Heat transfer characteristics of water flowing through microchannels, *Exp. Heat Transfer* 7 (1994) 265–283.
- [6] X.F. Peng, G.P. Peterson, Convective heat transfer and flow friction for water flow in microchannel structures, *Int. J. Heat Mass Transfer* 39 (1996) 2599–2608.
- [7] G.M. Mala, D. Li, Flow characteristics of water in microtubes, *Int. J. Heat Fluid Flow* 20 (1999) 142–148.
- [8] M. Faghri, S.E. Turner, Gas flow and heat transfer in microchannels, in: *Proc. of SAREK Summer Annual Conference*, Muju, Korea, 2003, pp. 542–550.
- [9] X. Tu, P. Hrnjak, Experimental investigation of single-phase flow pressure drop through rectangular microchannels, in: *1st International Conference on Microchannels and Minichannels*, Rochester, NY, 2003.
- [10] D. Liu, S.V. Garimella, Investigation of liquid flow in microchannels, *AIAA J. Thermophys. Heat Transfer* 18 (2004) 65–72.
- [11] D. Yu, R. Warrington, R. Barron, T. Ameel, An experimental and theoretical investigation of fluid flow and heat transfer in microtubes, *ASME/JSME Therm. Eng. Conf.* 1 (1995) 523–530.
- [12] H.J. Lee, S.Y. Lee, Pressure drop correlations for two-phase flow within horizontal rectangular channels with small heights, *Int. J. Multiphase Flow* 27 (2001) 783–796.
- [13] E.W. Lemmon, M.O. McLinden, M.L. Huber, NIST Standard Reference Database 23, NIST reference fluid thermodynamic and transport properties—REFPROP, Version 7.0, National Institute of Standards and Technology, Boulder, CO, 2002.
- [14] H.W. Coleman, W.G. Steele, *Experimentation and Uncertainty Analysis for Engineers*, John Wiley & Sons, New York, 1989.
- [15] N.T. Obot, Toward a better understanding of friction and heat/mass transfer in microchannels—a literature review, *Microscale Thermophys. Eng.* 6 (2002) 155–173.
- [16] R.W. Lockhart, R.C. Martinelli, Proposed correlation of data for isothermal two-phase two-component flow in pipes, *Chem. Eng. Prog.* 45 (1949) 39.
- [17] K. Moriyama, A. Inoue, H. Ohira, The thermohydraulic characteristics of two-phase flow in extremely narrow channels (the frictional pressure drop and void fraction of adiabatic two-component two-phase flow), *Trans. JSME (Ser. B)* 58 (1992) 401–407.
- [18] K. Mishima, T. Hibiki, Some characteristics of air–water two-phase flow in small diameter vertical tubes, *Int. J. Multiphase Flow* 22 (1996) 703–712.
- [19] P.A. Kew, K. Cornwell, Correlations for the prediction of boiling heat transfer in small-diameter channels, *Appl. Therm. Eng.* 17 (1997) 705–715.